

Characterization of circular D0L systems

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Abstract

We prove that every non-circular D0L system contains arbitrarily long repetitions. This result was already published in 1993 by Mignosi and Séébold, however their proof is only a sketch. We give here a complete proof. Further, employing our previous result, we give a simple algorithm to test circularity of an injective D0L system.

Keywords: D0L system, circular D0L system, repetition, critical exponent
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1. Introduction

In formal language theory, D0L languages form an important class. See for instance [RS80]. Starting by the work of Axel Thue, repetitions in various languages were studied. In [ER83], the authors show that it is decidable whether a D0L language is k -power free, i.e., does not contain a repetition of k same words for some $k \in \mathbb{N}$. In [MS93], the authors show that if a PD0L language is k -power free for some integer k , then it is circular. However, the authors give mostly only sketches of proofs, thus we give a sound proof here. Moreover, we generalize the result as we prove it for non-injective PD0L-systems and slightly relaxed definition of circularity, called weak circularity. We also give a simple algorithm to test whether an injective D0L system is circular.

2. Preliminaries

Let \mathcal{A} be an *alphabet*: a finite set of *letters*. The free monoid \mathcal{A}^* is the set of all finite words over \mathcal{A} endowed with concatenation. The *empty word* is denoted ε and the set of all non-empty words over \mathcal{A} is denoted \mathcal{A}^+ . The *length* of $w \in \mathcal{A}^*$ is denoted $|w|$. Given a word $w \in \mathcal{A}^*$, we say that $u \in \mathcal{A}^*$ is a *factor* of w if there exists words p and s , possibly empty, such that $w = pus$. Such a word p is a *prefix* of w , and the word s is a *suffix* of w . If $|p| < |w|$, p is a *proper* prefix, if $|s| < |w|$, s is a *proper* suffix.

The set $\mathcal{A}^{\mathbb{N}}$ is the set of all *infinite words* over \mathcal{A} . Given a word w , by w^ω we denote the infinite word $www \cdots$.

Let φ be an endomorphism of \mathcal{A}^* . We define

$$\|\varphi\| = \max\{|\varphi(a)| : a \in \mathcal{A}\} \quad \text{and} \quad |\varphi| = \min\{|\varphi(a)| : a \in \mathcal{A}\}.$$

A triplet $G = (\mathcal{A}, \varphi, w)$ is a *D0L system* if \mathcal{A} is an alphabet, φ is an endomorphism of \mathcal{A}^* , and $w \in \mathcal{A}^*$. The word w is the *axiom* of G . The *sequence* of G is $E(G) = (w_i)_{i \geq 0}$ where $w_0 = w$ and $w_i = \varphi^i(w_0)$. The *language* of G is the set $L(G) = \{\varphi^n(w) : n \in \mathbb{N}\}$ and by $S(L(G))$ we denote the set of all factors appearing in $L(G)$. The alphabet is always considered to be the minimal alphabet necessary, i.e., $\mathcal{A} \cap S(L(G)) = \mathcal{A}$.

We say that a D0L system $G = (\mathcal{A}, \varphi, w)$ is *injective* if for every $w, v \in S(L(G))$, $\varphi(w) = \varphi(v)$ implies that $w = v$. It is clear that if φ is injective, then G is injective. The converse is not true: consider $\varphi : a \rightarrow abc, b \rightarrow bc, c \rightarrow a$, then φ is not injective as $\varphi(cb) = \varphi(a)$ but $G = (\{a, b, c\}, \varphi, a)$ is injective since $cb \notin S(L(G))$. If φ is *non-erasing*, i.e., $\varphi(a) \neq \varepsilon$ for all $a \in \mathcal{A}$, then we speak about *propagating D0L system*, shortly PD0L.

Given a D0L system $G = (\mathcal{A}, \varphi, w)$ we say that the letter a is *bounded* (or also of *rank zero*) if the set $\{\varphi^n(a) : n \in \mathbb{N}\}$ is finite. If a letter is not bounded, it is *unbounded*. We denote the set of all bounded letters by \mathcal{A}_0 . The system G is *pushy* if $S(L(G))$ contains infinitely many factors over \mathcal{A}_0 .

A D0L system is *repetitive* if for any $k \in \mathbb{N}$ there is a non-empty word w such that w^k is a factor. By [ER83], any repetitive D0L system is *strongly repetitive*, i.e., there is a non-empty word w such that w^k is a factor for all $k \in \mathbb{N}$.

3. Definition of circularity

In the literature, one can find two slightly different views of circularity. Both these views can be expressed in terms of interpretations.

Definition 1. Let $G = (\mathcal{A}, \varphi, w)$ be a PD0L-system. A triplet (p, v, s) where $p, s \in \mathcal{A}^*$ and $v = v_1 \cdots v_n \in \mathcal{A}^+$ is an *interpretation* of a word $u \in S(L(G))$ if $\varphi(v) = pus$.

The following definition of circularity is used in [MS93].

Definition 2. Let $G = (\mathcal{A}, \varphi, w)$ be a PD0L-system and let (p, v, s) and (p', v', s') be two interpretations of a non-empty word $u \in S(L(G))$ with $v = v_1 \cdots v_n$, $v' = v'_0 \cdots v'_m$ and $u = u_1 \cdots u_\ell$.

We say that G is *circular* with synchronization delay $D > 0$ if whenever we have

$$|\varphi(v_1 \cdots v_i)| - |p| > D \quad \text{and} \quad |\varphi(v_{i+1} \cdots v_n)| - |s| > D$$

for some $1 \leq i \leq n$, then there is $1 \leq j \leq m$ such that

$$|\varphi(v_1 \cdots v_{i-1})| - |p| = |\varphi(v'_1 \cdots v'_{j-1})| - |p'|$$

and $v_i = v'_j$ (see Figure 1).

This definition says that a long enough word has unique φ -preimage except for some prefix and suffix shorter than a constant D . Note that if a D0L system $G = (\mathcal{A}, \varphi, w)$ contains arbitrarily long words with two different φ -preimages (i.e., for any $n > 0$ there are words v and u in $S(L(G))$ longer than n with $\varphi(v) = \varphi(u)$) cannot be circular.

In [Cas94], a circular D0L system with injective morphism is defined using the notion of synchronizing point (see Section 3.2 in [Cas94] for details). We give here an equivalent definition employing the notion of interpretation.

Definition 3. Let $G = (\mathcal{A}, \varphi, w)$ be a PD0L-system. We say that two interpretations (p, v, s) and (p', v', s') of a word $u \in SL(G)$ are synchronized at position k if there exist nonnegative indices i and j such that

$$\varphi(v_1 \cdots v_i) = pu_1 \cdots u_k \quad \text{and} \quad \varphi(v'_1 \cdots v'_j) = p'u_1 \cdots u_k$$

with $v = v_1 \cdots v_n$, $v' = v'_0 \cdots v'_m$ and $u = u_1 \cdots u_\ell$ (see Figure 2).

We say that a word $u \in S(L(G))$ has a synchronizing point at position k with $0 \leq k \leq |u|$ if all its interpretations are pairwise synchronized at position k .

By [Cas94], a D0L system G with injective morphism is circular if there is positive D such that any v from $S(L(G))$ longer than $2D$ has a synchronizing point. This definition is equivalent to Definition 2. However, the synchronizing point is defined for D0L systems with just non-erasing morphism and so we can omit the assumption of injectiveness in Definition 2.

Definition 4. A PD0L-system G is called weakly circular if there is a constant $D > 0$ such that any v from $S(L(G))$ longer than $2D$ has a synchronizing point.

As said above, if G is injective, weak circularity is equivalent to circularity. As the following example shows, this is not true for the non-injective case.

Example 5. Consider the D0L system $G_1 = (\{a, b, c\}, \varphi_1, a)$ with the non-injective $\varphi_1 : a \rightarrow abca, b \rightarrow bc, c \rightarrow bc$. This system is not circular as for all $m \in \mathbb{N}$ the word $(bc)^{2m}$ has two different preimages $(bc)^m$ and $(cb)^m$. The corresponding interpretations, however, have synchronizing points for $m > 1$ at positions $2k$ for all $0 \leq k \leq m$. Moreover, one can easily check that G_1 is weakly circular.

So, circularity implies weak circularity but the converse is not true.

4. Main result

Theorem 6. Any PD0L system that is not weakly circular is repetitive.

The two following lemmas will be used to prove this theorem. The next lemma and its proof is based on the ideas in the proof of Theorem 4.35 in [Kür03].

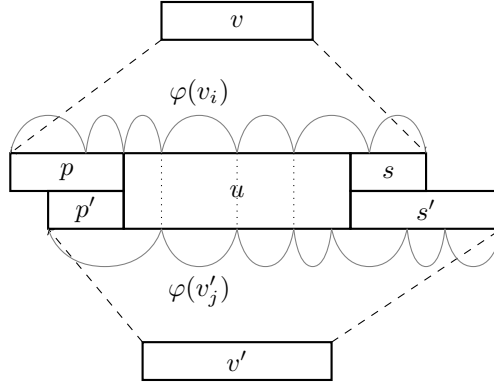


Figure 1: Two interpretations from Definition 2 with $v_i = v'_j$.

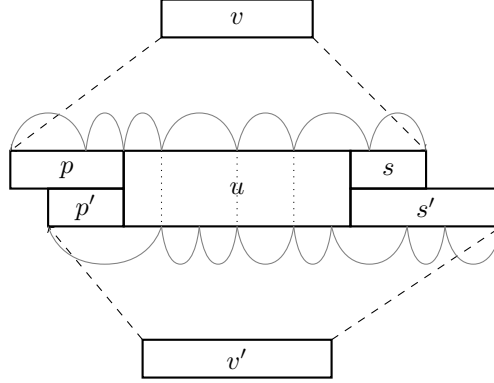


Figure 2: Two interpretations from Definition 3 synchronized at positions depicted by dotted lines.

Lemma 7. *Let $G = (\mathcal{A}, \varphi, w)$ be a PDOL system. If there exists a sequence $\epsilon(k)$ with $\lim_{k \rightarrow +\infty} \epsilon(k) = +\infty$ and if for any $k \in \mathbb{N}$ there are two non-empty words u and v in $S(L(G))$ containing an unbounded letter such that the following conditions are satisfied*

- (i) $|u| = k$;
- (ii) *there are two integers m and n such that $m > n$ and letters a and b such that for each $i \in \{m, n\}$ the word $\varphi^i(u)$ is a factor of $\varphi^i(v)$ and $\varphi^i(v)$ is a factor of $\varphi^i(aub)$, moreover, $\frac{|\varphi^i(u)|}{|\varphi^i(a)|} > \epsilon(k)$ or $\frac{|\varphi^i(u)|}{|\varphi^i(b)|} > \epsilon(k)$; and*
- (iii) *for each $i \in \{m, n\}$ the factor $\varphi^i(u)$ has no synchronizing point: two non-synchronized interpretations are $(\varepsilon, \varphi^{i-1}(u), \varepsilon)$ and $(p_i, \varphi^{i-1}(v), s_i)$,*

then the DOL system is repetitive.

Proof. Suppose that $\frac{|\varphi^i(u)|}{|\varphi^i(a)|} > \epsilon(k)$ is true in requirement (ii), the other case $\frac{|\varphi^i(u)|}{|\varphi^i(b)|} > \epsilon(k)$ is analogous. It holds that

$$\varphi^m(v) = p_m \varphi^m(u) s_m = \varphi^{m-n}(\varphi^n(v)) = \varphi^{m-n}(p_n) \varphi^m(u) \varphi^{m-n}(s_n).$$

The fact that the interpretations $(\varepsilon, \varphi^{m-1}(u), \varepsilon)$ and $(p_m, \varphi^{m-1}(v), s_m)$ are not synchronized implies that $p_m \neq \varphi^{m-n}(p_n)$ (if $p_m = \varphi^{m-n}(p_n)$, the two interpretations of $\varphi^m(u)$ are synchronized at position 0, see Figure 3). Since $p_m \varphi^m(u) s_m = \varphi^{m-n}(p_n) \varphi^m(u) \varphi^{m-n}(s_n)$ the word p_m is a proper prefix of $\varphi^{m-n}(p_n)$ or vice versa. Moreover, p_m is not empty since it contradicts again the point (iii). Suppose p_m is a non-empty proper prefix of $\varphi^{m-n}(p_n)$. It implies there exists a word z such that $p_m z = \varphi^{m-n}(p_n)$. If $\varphi^{m-n}(p_n)$ is a non-empty proper prefix of p_m , then we may find a word z such that $\varphi^{m-n}(p_n) z = p_m$ (see Figure 4).

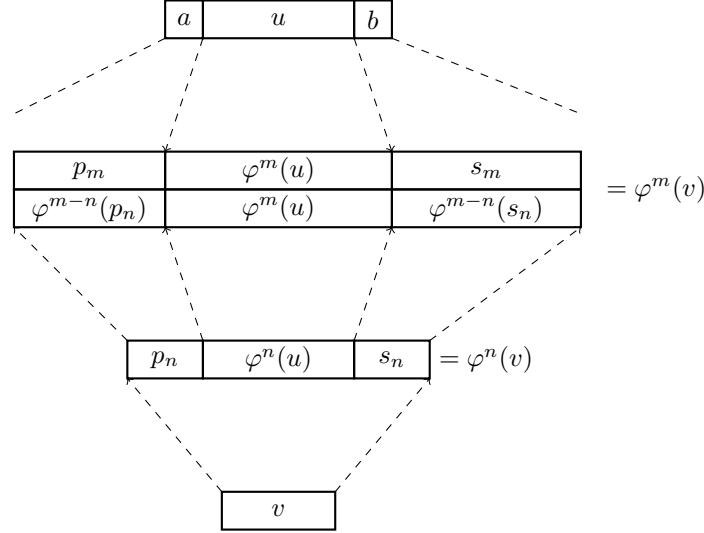


Figure 3: The first arrangement from the proof of Lemma 7.

Therefore, in both cases, the word $\varphi^m(u)$ is a prefix of z^ℓ for some integer ℓ . Since

$$\frac{|\varphi^m(u)|}{|z|} > \frac{|\varphi^m(u)|}{\max\{|p_m|, |\varphi^{m-n}(p_n)|\}} > \frac{|\varphi^m(u)|}{|\varphi^m(a)|} > \epsilon(k),$$

we deduce that $z^{\lfloor \epsilon(k) \rfloor}$ is factor of $\varphi^m(u)$.

As $\lim_{k \rightarrow +\infty} \epsilon(k) = +\infty$, the D0L-system is repetitive. \square

Lemma 8. *In any PD0L system there is a constant C such that all factors over bounded letters longer than C have a synchronizing point.*

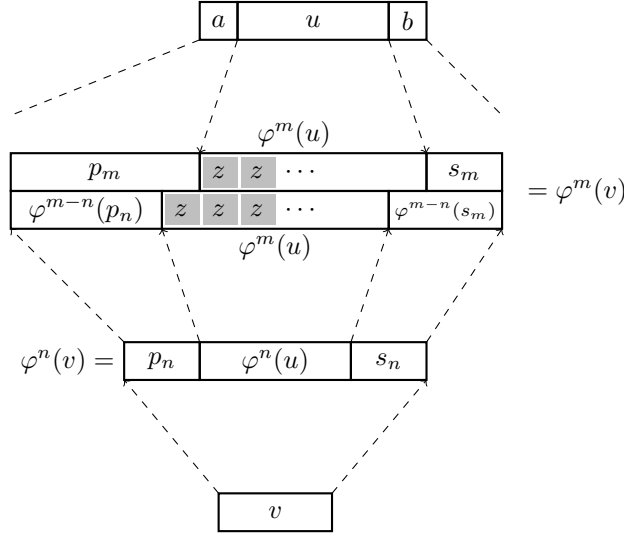


Figure 4: The second arrangement from the proof of Lemma 7.

Proof. The statement is trivial for non-pushy D0L systems, hence we consider a pushy one. Clearly, there exist an integer n such that for all $c \in \mathcal{A}_0$ we have $|\varphi^m(c)| = |\varphi^{m+1}(c)|$ for every $m \geq n$. Let u be a factor over bounded letters only of length at least $L = 3\|\varphi^{n+1}\| \cdot |w_0|$ where w_0 is the axiom of the D0L system. This implies that u appears in the sequence $E(G) = (w_i)_{i \geq 0}$ in w_k for $k > n + 1$.

Let (p, w, s) be an interpretation of u . Since u is a factor of w_k such that $k > n + 1$ and $|w_k| > L$, there must be words $x, y \in \mathcal{A}^*$ and $v \in \mathcal{A}^+$ such that $w = x\varphi^n(v)y$ and

$$|\varphi(x)| - |p| < \|\varphi^{n+1}\| \quad \text{and} \quad |\varphi(y)| - |s| < \|\varphi^{n+1}\|.$$

As $\varphi^{n+1}(v)$ is a factor of u , it contains only bounded letters, and thus so does the word v . Moreover, by the definition of n , every letter c occurring in $\varphi^n(v)$ satisfies $|\varphi^n(c)| = |\varphi^{n+1}(c)|$.

It follows that any two interpretations (p, w, s) and (p', w', s') of the word u are synchronized at position $k = \|\varphi^{n+1}\|$. \square

Proof of Theorem 6. Consider a PD0L system $G = (\mathcal{A}, \varphi, w_0)$ with infinite language (the statement for D0L system with finite language is trivial). We define a partition of the alphabet $\mathcal{A} = \Sigma_m \cup \Sigma_{m-1} \cup \dots \cup \Sigma_1 \cup \Sigma_0$ as follows:

- (i) $\Sigma_0 = \mathcal{A}_0$ is the set of bounded letters,
- (ii) if x and y are from Σ_i , then the sequence $\left(\frac{|\varphi^n(x)|}{|\varphi^n(y)|} \right)_{n \geq 1}$ is $\Theta(1)$,

(iii) for all $i = 1, \dots, m$, if x is an element of Σ_i and y of Σ_{i-1} , then $\lim_{n \rightarrow +\infty} \frac{|\varphi^n(x)|}{|\varphi^n(y)|} = +\infty$.

This partition is well defined due to [SS78] where it is proved that for any $a \in \mathcal{A}$ there are numbers $\alpha \in \mathbb{N}$ and $\beta \in \mathbb{R}_{\geq 1} \cup \{0\}$ such that $|\varphi^n(a)| = \Theta(n^\alpha \beta^n)$. Further we define for all $j = 0, 1, \dots, m$ the sets

$$\mathcal{A}_j = \bigcup_{0 \leq i \leq j} \Sigma_i.$$

Note that $\varphi(\mathcal{A}_j) \subset \mathcal{A}_j$ and $\varphi(\mathcal{A}_j) \cap \Sigma_j \neq \emptyset$.

Lemma 8 implies that factors without synchronizing point over \mathcal{A}_0 are bounded in length. Fix a positive integer j and assume that there is a factor without synchronizing point of arbitrary length over \mathcal{A}_j . Let k be a positive integer. For any positive $\ell \in \mathbb{N}$ we can find words $u_\ell^{(k)} \in \mathcal{A}_j^*$ and $v_\ell^{(k)} \in \mathcal{A}_j^*$ and letters $a_\ell^{(k)} \in \mathcal{A}_j$ and $b_\ell^{(k)} \in \mathcal{A}_j$ such that

- (a) $|u_\ell^{(k)}| = k$,
- (b) $\varphi^\ell(v_\ell^{(k)})$ is a factor of $\varphi^\ell(a_\ell^{(k)} u_\ell^{(k)} b_\ell^{(k)})$ and $\varphi^\ell(u_\ell^{(k)})$ is a factor of $\varphi^\ell(v_\ell^{(k)})$,
- (c) $\varphi^\ell(u_\ell^{(k)})$ has two non-synchronized interpretations

$$(\varepsilon, \varphi^{\ell-1}(u_\ell^{(k)}), \varepsilon) \text{ and } (p_\ell^{(k)}, \varphi^{\ell-1}(v_\ell^{(k)}), s_\ell^{(k)})$$

where $p_\ell^{(k)} \varphi^\ell(u_\ell^{(k)}) s_\ell^{(k)} = \varphi^\ell(v_\ell^{(k)})$.

Since the length of $u_\ell^{(k)}$ is fixed, there must be an infinite set $E_1^{(k)} \subset \mathbb{N}$ such that $u_i^{(k)} = u_j^{(k)} = u^{(k)}$, $a_i^{(k)} = a_j^{(k)} = a^{(k)}$ and $b_i^{(k)} = b_j^{(k)} = b^{(k)}$ for all i, j from $E_1^{(k)}$.

If for each k there are indices $\ell_1 > \ell_2$ in $E_1^{(k)}$ such that $v_{\ell_1}^{(k)} = v_{\ell_2}^{(k)} = v^{(k)}$ and if the number of letters from Σ_j in $u^{(k)}$ tends to $+\infty$ as $k \rightarrow +\infty$, then G is repetitive by Lemma 7 and the proof is finished.

Assume no such indices ℓ_1, ℓ_2 exist for some k , then $|v_\ell^{(k)}|$ must go to infinity as $\ell \rightarrow +\infty$. It follows from (a) and (b) that the number of letters from Σ_j in words $v_\ell^{(k)}$ is bounded (or even zero) and so there must be $j' \in \{1, \dots, j-1\}$ such that the number of letters from $\Sigma_{j'}$ in $v_\ell^{(k)}$ goes to infinity as $\ell \rightarrow +\infty$ and there is a factor without a synchronizing point over $\mathcal{A}_{j'}$ of arbitrary length. Note that such j' must exist since number of letters from Σ_j is bounded and the factors of $v_\ell^{(k)}$ containing only letters from \mathcal{A}_0 are bounded in length.

If such indices ℓ_1, ℓ_2 exist for each k but the number of letters from Σ_j in $u^{(k)}$ is bounded as $k \rightarrow +\infty$, there must be again some $j' \in \{1, \dots, j-1\}$ such that the number of letters from $\Sigma_{j'}$ in $u^{(k)}$ goes to infinity as $k \rightarrow +\infty$ and there is again a factor without a synchronizing point over $\mathcal{A}_{j'}$ of arbitrary length.

Overall, given the integer j , we either prove G is repetitive by Lemma 7 or we find a positive integer j' less than j such that there is a factor without a synchronizing point over $\mathcal{A}_{j'}$ of arbitrary length. In the latter case we repeat the construction for $j = j'$.

The only remaining case is when $j = 1$, i.e. we have a factor without a synchronizing point over \mathcal{A}_1 of arbitrary length. Even in this case we can repeat the construction above. However, the case when $\ell_1 > \ell_2$ such that $v_{\ell_1}^{(k)} = v_{\ell_1}^{(k)} = v^{(k)}$ do not exists for some k is not possible.

Indeed, it cannot happen that $|v_{\ell}^{(k)}|$ goes to infinity as $\ell \rightarrow +\infty$: $v_{\ell}^{(k)}$ must consist of letters from $\mathcal{A}_1 = \Sigma_1 \cup \Sigma_0$. Since $u^{(k)}$ is over \mathcal{A}_1 as well (with at least one letter from Σ_1 for k large enough), the number of letters from Σ_1 in $v^{(k)}$ cannot be unbounded (for $\ell \rightarrow +\infty$) by the definition of Σ_1 . Clearly, again by Lemma 8, the number of letters from \mathcal{A}_0 in $v_{\ell}^{(k)}$ is bounded as well (for $\ell \rightarrow +\infty$) and so the indices $\ell_1 > \ell_2$ must exist so that $v_{\ell_1}^{(k)} = v_{\ell_1}^{(k)} = v^{(k)}$.

Moreover, since factors without a synchronizing point over \mathcal{A}_0 are bounded in length, the number of letters from Σ_1 in $u^{(k)}$ goes to infinity as $k \rightarrow +\infty$.

This all implies that G is repetitive by Lemma 7. □

5. Simple criterion for circularity

Definition 9. We say that a D0L system G is unboundedly repetitive if there exists $w \in S(L(G))$ such that $w^k \in S(L(G))$ for all k and w contains at least one unbounded letter.

In [ER78], the authors introduced the notion of simplification to study properties of a D0L system. Given an endomorphism φ over \mathcal{A} , the endomorphism Ψ over \mathcal{B} is its *simplification* if $\#\mathcal{B} < \#\mathcal{A}$ and there exist morphisms $h : \mathcal{A}^* \rightarrow \mathcal{B}^*$ and $k : \mathcal{B}^* \rightarrow \mathcal{A}^*$ such that $\varphi = kh$ and $\Psi = hk$. A corollary of the defect theorem (see [KO00]) is that every non-injective morphism has a simplification which is injective, called an *injective simplification*. Specially, injective G is its own injective simplification.

The following claim follows from Proposition 4.3 in [KO00] and Theorem 2 in [KtS13].

Proposition 10. A D0L system G is unboundedly repetitive if and only if for some its injective simplification $G' = (\mathcal{B}, \psi, w'_0)$ of G there is a positive integer ℓ and $a \in \mathcal{B}$ such that

$$(\psi^{\ell})^{\infty}(a) = w^{\omega} \quad \text{for some } w \in \mathcal{B}^+.$$

In fact, if the condition in the previous claim is satisfied for some injective simplification, then it is satisfied for all injective simplifications.

Using this proposition and Theorem 1 of [KtS13] we deduce the following theorem.

Theorem 11. *Let G be a repetitive D0L system, then one of the following is true:*

- (i) G is pushy,
- (ii) G is unboundedly repetitive.

In the previous section we proved that any PD0L system that is not weakly circular is repetitive. The next theorem gives a characterization of injective circular D0L systems.

Theorem 12. *An injective D0L system $G = (\mathcal{A}, \varphi, w)$ is not circular if and only if it is unboundedly repetitive.*

Proof. (\Rightarrow): As an injective morphism is non-erasing, Theorem 6 implies that G is repetitive. Thus, by Theorem 11, G is pushy or unboundedly repetitive. Suppose it is pushy and not unboundedly repetitive. Therefore, there exist an integer N such that all repetitions u^ℓ where $\ell > N$ and $u \in S(L(G))$ are over bounded letters only, i.e., $u \in \mathcal{A}_0^+$. From the proof of Theorem 6 one can see that long enough non-synchronized factors contain longer and longer repetitions but these repetitions cannot be over bounded letters due to Lemma 8 – a contradiction.

(\Leftarrow): Proposition 10 implies that there is a positive integer ℓ and a letter a such that $(\varphi^\ell)^\infty(a) = w^\omega$ for some $w \in \mathcal{A}^+$. In [KtS13] it is proved that the word w can be taken so that it contains the letter a only once at its beginning. It follows that $\varphi^\ell(w) = w^k$ for some $k > 1$. Since φ is injective, we must have $\varphi(p) \neq w$ for all prefixes p of w . This implies that for all $n \in \mathbb{N}$ the word w^{nk} has two non-synchronized interpretations $(\varepsilon, w^n, \varepsilon)$ and (w, w^{n+1}, w^{k-1}) . \square

Remark 13. In the previous theorem, we cannot omit assumption of injectiveness and replace circularity with weak circularity: consider again the D0L system G_1 from example 5. The conditions of Proposition 10 is satisfied for $\ell = 1$ and the letter b with $w = bc$ but still the corresponding D0L system is weakly circular.

Since the existence of ℓ and a satisfying conditions of Proposition 10 can be tested by a simple and fast algorithm described in [Lan91], we have a simple algorithm deciding circularity.

As a corollary of Theorem 12, we retrieve the following result of [Mos96]. A morphism $\varphi : \mathcal{A}^* \rightarrow \mathcal{A}^*$ is *primitive* if there exists an integer k such that for all letters $a, b \in \mathcal{A}$, the letter b appears in $\varphi^k(a)$. An infinite word \mathbf{u} is a *periodic point* of a morphism φ if there exists an integer k such that $\varphi^k(\mathbf{u}) = \mathbf{u}$.

Corollary 14 ([Mos96]). *If \mathbf{u} is an aperiodic fixed point of a primitive morphism injective on $S(L(G))$, then it is circular.*

Proof. Any periodic point of a primitive morphism has the same language as \mathbf{u} . Therefore, every periodic point is aperiodic and so the condition of Proposition 10 cannot be satisfied. Theorem 12 yields the result. \square

Acknowledgements

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